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FDTD Modeling
of
Thin Impedance Sheets

by

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Abstract

Thin sheets of resistive or dielectric material are commonly encountered in radar cross section calculations. Analysis of such sheets is simplified by using sheet impedances. In this paper it is shown that sheet impedances can be modeled easily and accurately using Finite Difference Time Domain (FDTD) methods.

Introduction

In [1] a review of various approximate boundary conditions is given, including several for thin sheets and layers. These are applicable to sheets which are thin relative to the free space wavelength, so that they can be approximated by an electric current sheet. If the thin sheet is primarily conductive the sheet impedance will be resistive, as is the case for resistance cards. A thin lossless dielectric sheet will have a purely reactive sheet impedance, while in general the sheet impedance will be complex. These sheets are characterized by a discontinuity in the tangential magnetic field on either side of the sheet but no discontinuity in tangential electric field. This continuity, or single valued behavior of the electric field, allows the sheet current to be expressed in terms of an impedance multiplying this electric field.

Approach

The sheet impedance can be defined in several ways. A convenient definition can be obtained by combining eqs. (3.3) and (3.5) of [2]

$$Y_s = \sigma T + j\omega\epsilon_0(\epsilon_r - 1)T \quad (1)$$

with

$$Z_s = 1/Y_s \quad (2)$$

where Y_s is the sheet admittance, Z_s the sheet impedance, σ and ϵ_r the conductivity and relative permittivity of the sheet material, T the sheet thickness, and ϵ_0 the free space permittivity.

Let us now consider how to incorporate this approximation into the FDTD method. The surface impedance approximation

requires the impedance sheet to be small compared with the free space wavelength. In most FDTD calculations the FDTD cell size (Yee [3] cells are used here) must be on the order of $1/10$ wavelength or less for reasonably accurate results. Scattering from an infinitesimally thin perfectly conducting plate has been calculated by approximating the plate as being one FDTD cell thick with good results [4]. If it is assumed that the same approach can be applied to infinitesimally thin impedance sheets, then the plate thickness T in (1) merely becomes the thickness of the FDTD cell, and the conductivity and/or relative permittivity to be used in the FDTD calculations are merely adjusted in accordance with (1) to give the desired sheet impedance. Note that the FDTD cell dimension need not correspond to the thickness of the actual physical plate. The FDTD cell thickness is used only to determine the conductivity and relative permittivity of the FDTD electric field location so that the desired sheet impedance is approximated. Note also that, even if the wavelength in the material forming the impedance sheet is much smaller than a free space wavelength, the FDTD cell size need not be correspondingly reduced.

Demonstration

The first demonstration will consist of calculating the far zone backscatter from a 29×29 cm flat plate of sheet impedance $Z_s = 500 \Omega$. The FDTD calculations will use cubical Yee cells with 1 cm edges. Using $T = 1$ cm, the corresponding FDTD conductivity is $\sigma = 0.2$ S/m. The FDTD calculations shown in Figures 1-8 are all made with the plate modeled by setting the conductivity to 0.2 S/m for x and y polarized electric field locations corresponding to single z dimension index over a range of x and y dimension indices to model the plate. The FDTD approach used and the transformation to the far zone is described in [4]. The problem space size, orientation and position of the

plate, incident Gaussian pulse plane wave, and time step size are also consistent with those in [4].

Figure 1 shows the far zone backscattered electric field for a Gaussian pulsed plane wave normally incident on the plate. In Figure 2 this result is Fourier transformed, converted to cross section, and compared with results using the Method of Moments [2]. The agreement is quite good, with the approximately 20 dB reduction in radar cross section relative to a perfectly conducting plate of the same size [4] consistently predicted by both methods.

In Figures 3-8 the same plate geometry and composition is considered but for non-normal incidence. The plate is perpendicular to the z axis, with edges parallel to the x and y axes, and the plane wave is incident from $\theta=45^\circ$, $\phi=30^\circ$ degrees. Figures 3-5 show the co-polarized backscatter far zone electric field for ϕ and θ polarizations and the cross-polarized backscatter as well. In Figures 6-8 these time domain results are Fourier transformed and converted to radar cross section for comparison with Moment Method [2] results. Again the agreement is quite good, except at the highest frequencies considered. These results indicate that perhaps 12 cells/wavelength are required for good accuracy for off-normal incidence. Comparing the results in Figure 6 with those in Figure 5 of [4], it is clear that changing from a perfectly to a finitely conducting plate changes the scattering level and frequency behavior, and that the FDTD and Moment Method results agree quite well on these effects.

In Figure 9 both FDTD and Moment Method [4] results for scattering by a plate with a complex sheet impedance are shown. The sheet impedance is determined by applying eqs. (1,2) with conductivity 0.25 S/m, relative permittivity 3.0, and thickness 1 cm., corresponding to the FDTD parameters used. Again the plane

wave is a Gaussian pulse incident from $\theta=45$, $\phi=30$ degrees. The FDTD results agree with the Moment Results for frequencies up to about 12 cells/wavelength.

The final result is for a plate with edge treatment. For this demonstration a 21 x 21 cm thin perfectly conducting plate is given a 4 cm border of sheet impedance $Z_s = 500 \Omega$, resulting in a square plate 29 x 29 cm. This edged plate is modeled in FDTD by setting x and y polarized electric field locations for a single z dimension index as being either perfect conductor for the central portion of the plate or with a conductivity of 0.2 S/m for the edges. The ESP4 calculations were made with a central perfectly conducting plate surrounded by 4 plates of sheet impedance $Z_s = 500 \Omega$ attached to the central plate using overlap modes. The results are compared in Figure 10 with excellent agreement between the two methods, both showing a significant difference due to the edge treatment when compared with the results of Figure 6.

Conclusions

The ability of the FDTD method to easily and accurately model scattering by sheet impedances was demonstrated by comparing FDTD results for scattering from flat plates modeled using sheet impedances with Method of Moment results. The approach described here is directly applicable to the Yee cell, and demonstrated good accuracy for frequencies up to approximately 12 cells per wavelength.

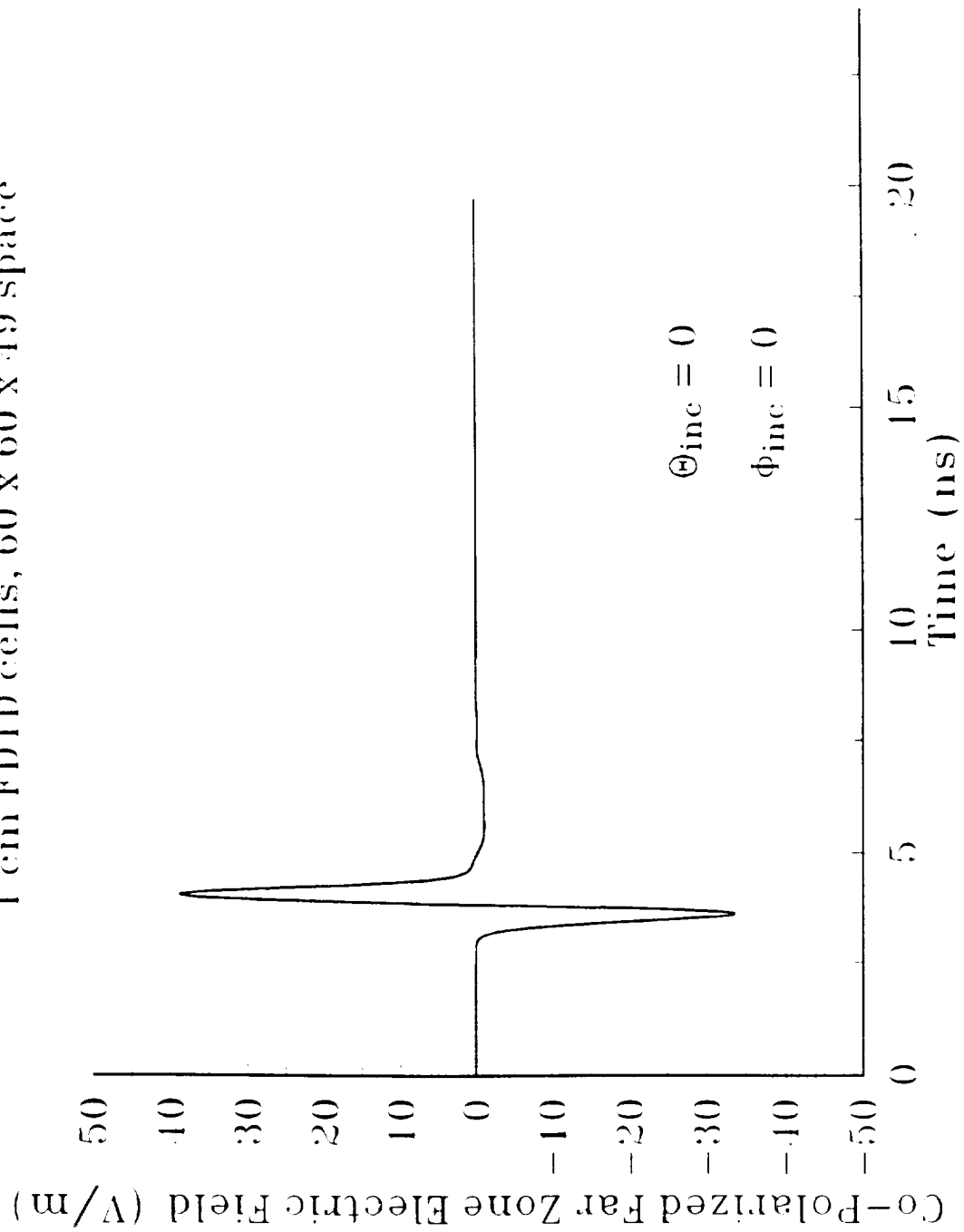
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2. E. Newman, "A user's manual for the electromagnetic surface patch code: ESP Version IV," The Ohio State University Research Foundation, ElectroScience Laboratory, Department of Electrical Engineering, Columbus, Ohio 43212.
3. K. S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media," IEEE Transactions on Antennas and Propagation, vol. AP-14, pp 302-307, May 1966.
4. R. Luebbers, K. Kunz, M. Schneider, F. Hunsberger, "A Finite Difference Time Domain Near-Zone to Far-Zone Transformation", IEEE Transactions on Antennas and Propagation, vol. AP-39, pp 429-433, April 1991.

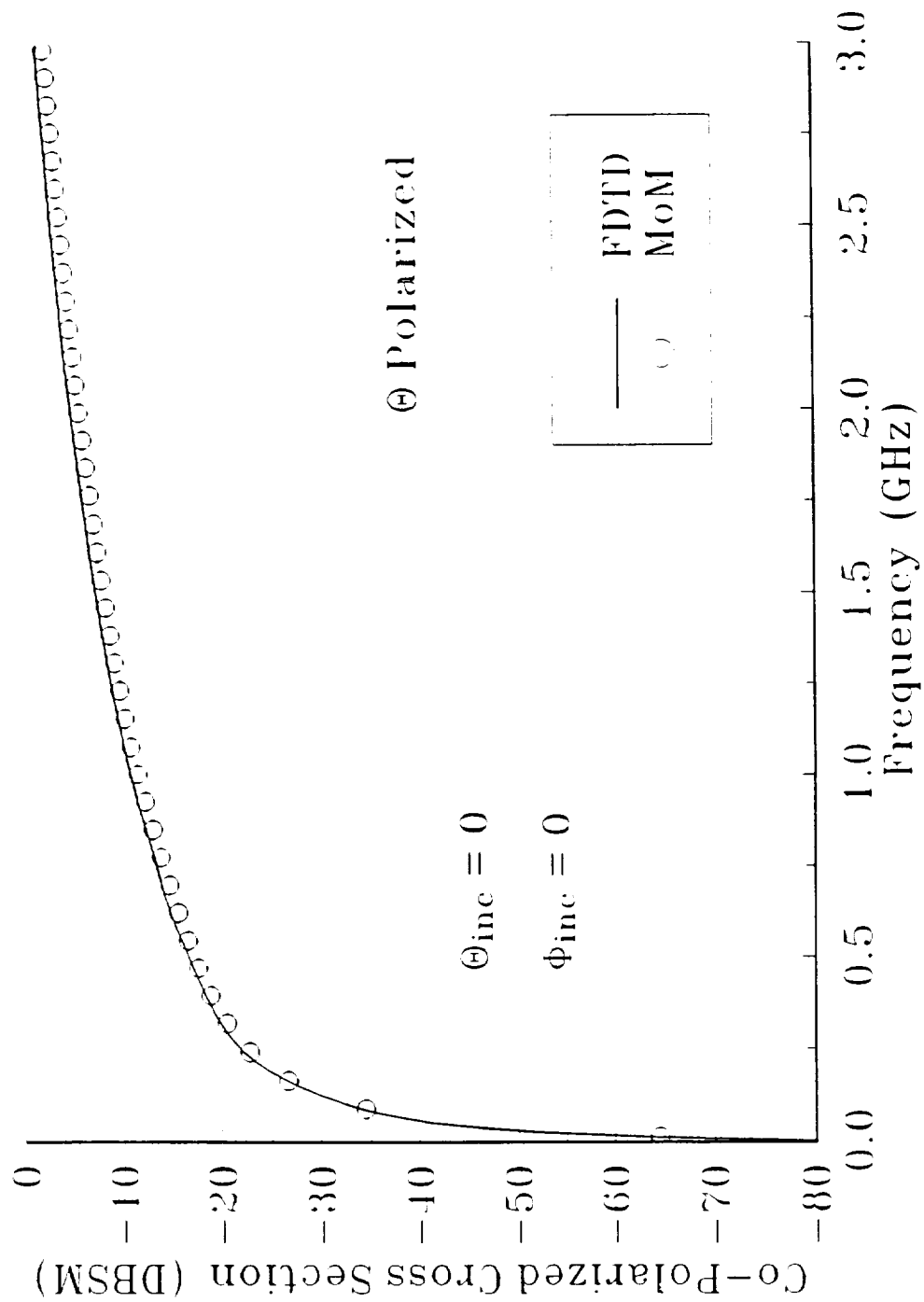
Figure Titles

1. Co-Polarized far zone electric field vs time scattered by a 29 x 29 cm flat plate of sheet impedance 500 ohms for a θ -polarized normally incident Gaussian pulse plane wave computed using FDTD.
2. Radar cross section for a 29 x 29 cm flat plate of sheet impedance 500 ohms, normal incidence, obtained from FDTD results of Figure 1 compared with Moment Method [2] results.
3. Co-Polarized far zone electric field vs time scattered by a 29 x 29 cm flat plate of sheet impedance 500 ohms for a ϕ -polarized incident Gaussian pulse plane wave from $\theta=45$, $\phi=30$ degrees computed using FDTD.
4. Co-Polarized far zone electric field vs time scattered by a 29 x 29 cm flat plate of sheet impedance 500 ohms for a θ -polarized incident Gaussian pulse plane wave from $\theta=45$, $\phi=30$ degrees computed using FDTD.
5. Cross-Polarized far zone electric field vs time scattered by a 29 x 29 cm flat plate of sheet impedance 500 ohms for a ϕ -polarized incident Gaussian pulse plane wave from $\theta=45$, $\phi=30$ degrees computed using FDTD.
6. Co-Polarized radar cross section for a 29 x 29 cm flat plate of sheet impedance 500 ohms, $\theta=45$, $\phi=30$ degree incidence, ϕ -polarized, obtained from FDTD results of Figure 3 compared with Moment Method [2] results.
7. Co-Polarized radar cross section for a 29 x 29 cm flat plate of sheet impedance 500 ohms, $\theta=45$, $\phi=30$ degree incidence, θ -polarized, obtained from FDTD results of Figure 4 compared with Moment Method [2] results.
8. Cross-Polarized radar cross section for a 29 x 29 cm flat plate of sheet impedance 500 ohms, $\theta=45$, $\phi=30$ degree incidence, obtained from FDTD results of Figure 5 compared with Moment Method [2] results.
9. Co-Polarized radar cross section for a 29 x 29 cm flat plate of sheet impedance corresponding to conductivity of 0.25, relative permittivity of 3.0, and thickness 1 cm, for $\theta=45$, $\phi=30$ degree ϕ -polarized incident plane wave calculated using FDTD and compared with Method of Moments [2].
10. Co-Polarized radar cross section for a 21 x 21 cm perfectly conducting flat plate with a 4 cm 500 ohm edge treatment on all sides (total plate size 29 x 29 cm) for $\theta=45$, $\phi=30$ degree ϕ -polarized incident plane wave calculated using FDTD and compared with Method of Moments [2].

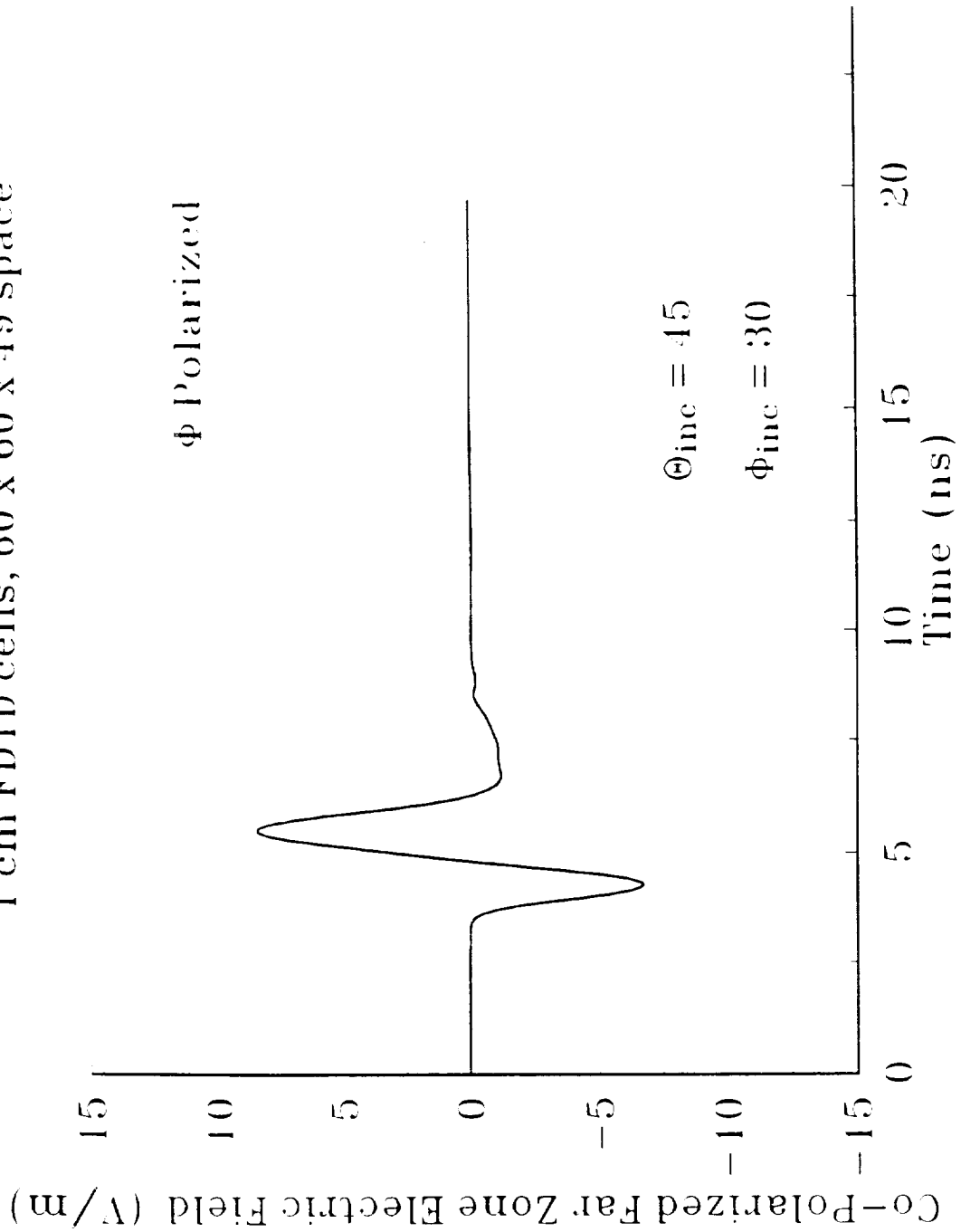
Flat Plate, 29 x 29 cm, $\sigma=0.2$ ($Z_s = 500 \Omega$)
1 cm FDTD cells, 60 x 60 x 49 space



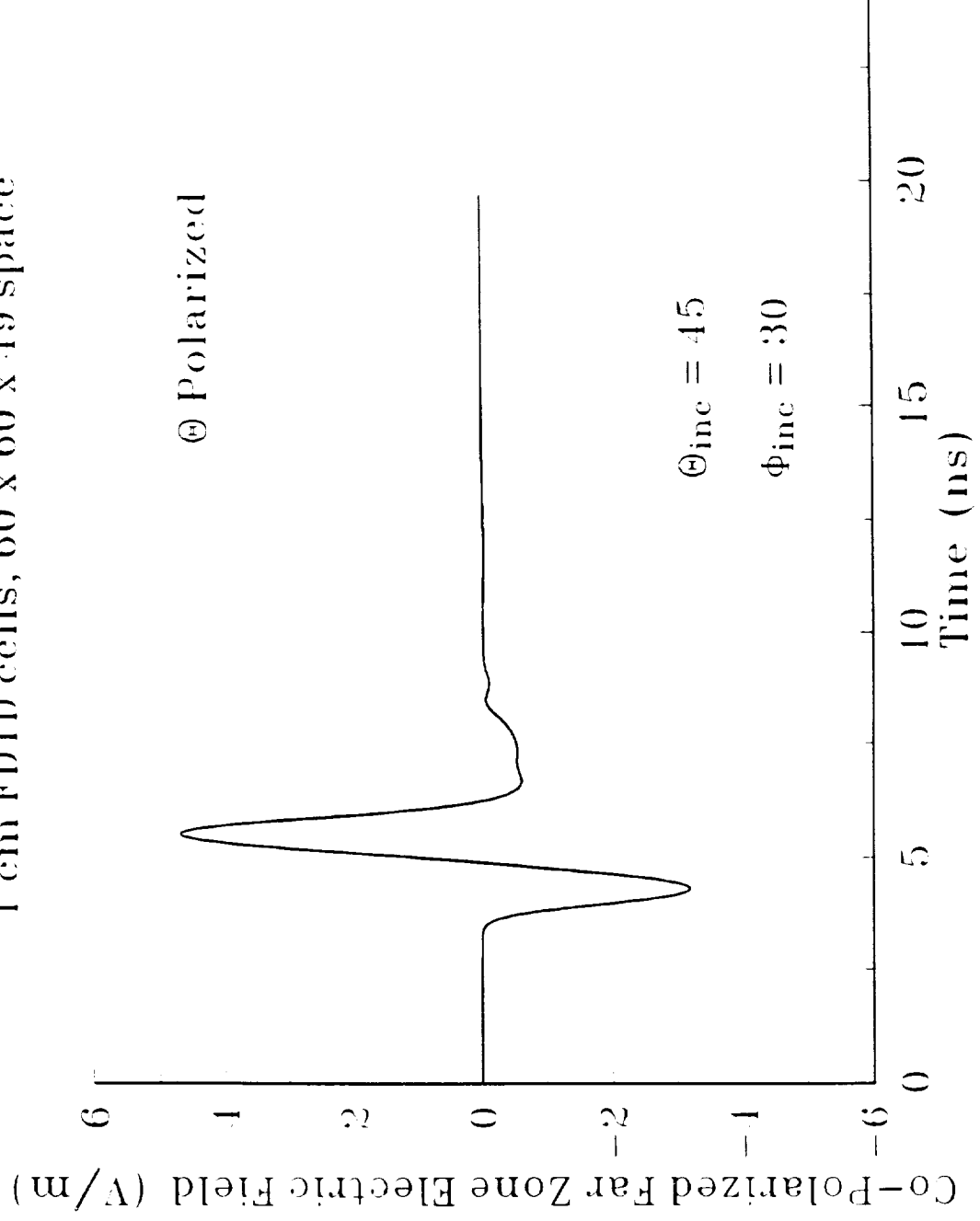
Flat Plate, 29 x 29 cm, $\sigma=0.2$ ($Z_s = 500 \Omega$)
 1 cm FDTD cells, 60 x 60 x 49 space



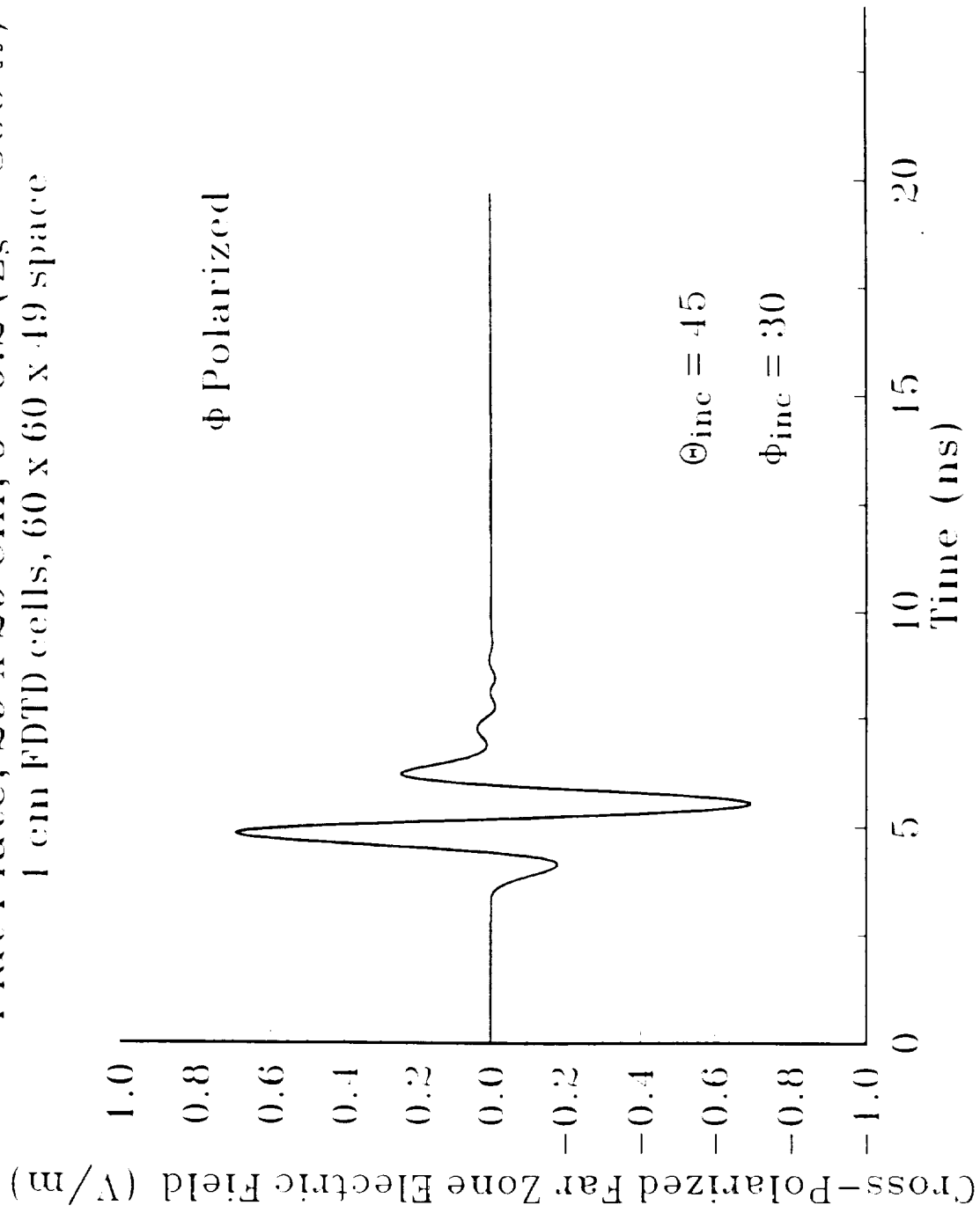
Flat Plate, 29 x 29 cm, $\sigma=0.2$ ($Z_s = 500 \Omega$)
 1 cm FDTD cells, 60 x 60 x 49 space



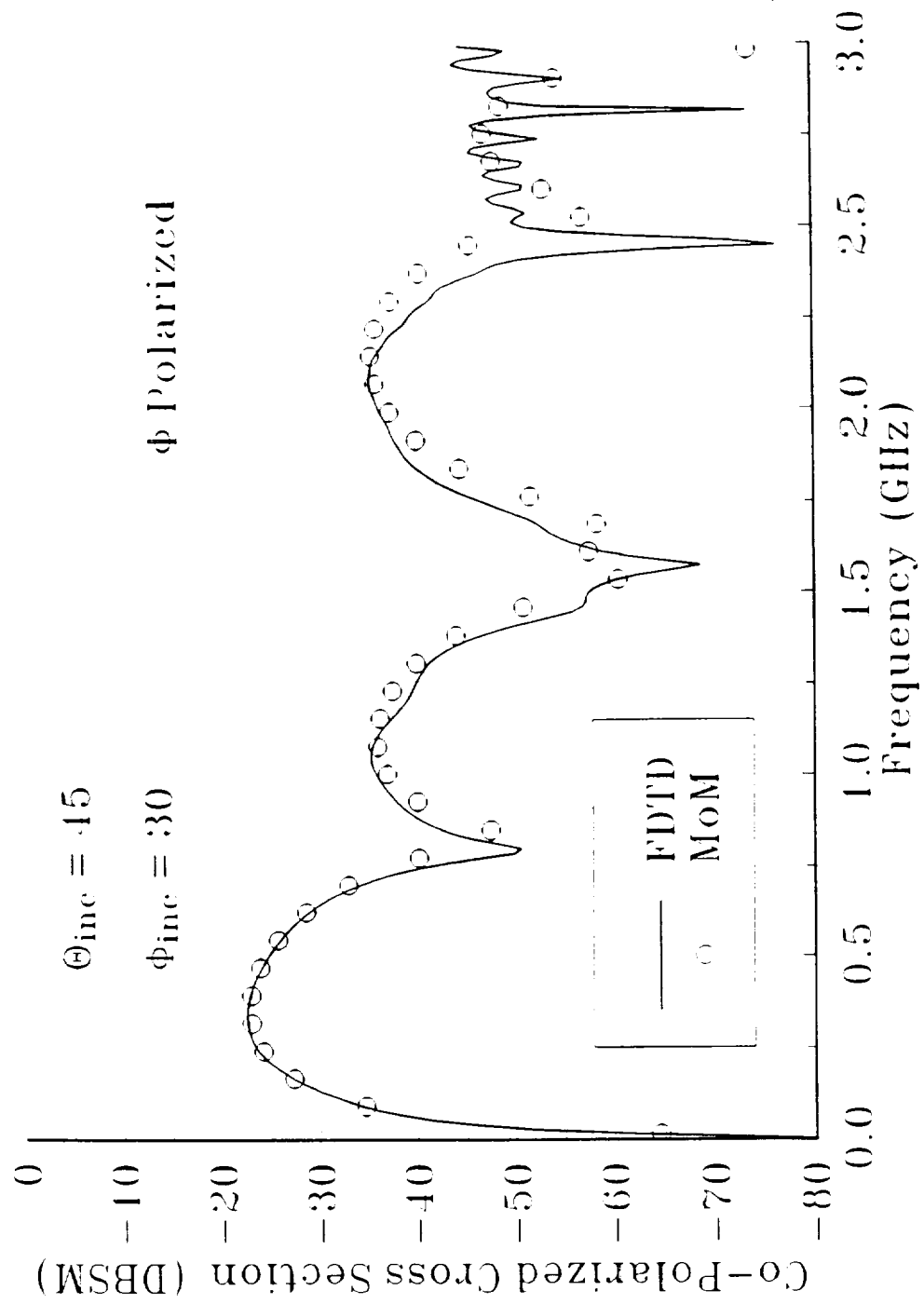
Flat Plate, 29 x 29 cm, $\sigma=0.2$ ($Z_s = 500 \Omega$)
 1 cm FDTD cells, 60 x 60 x 19 space



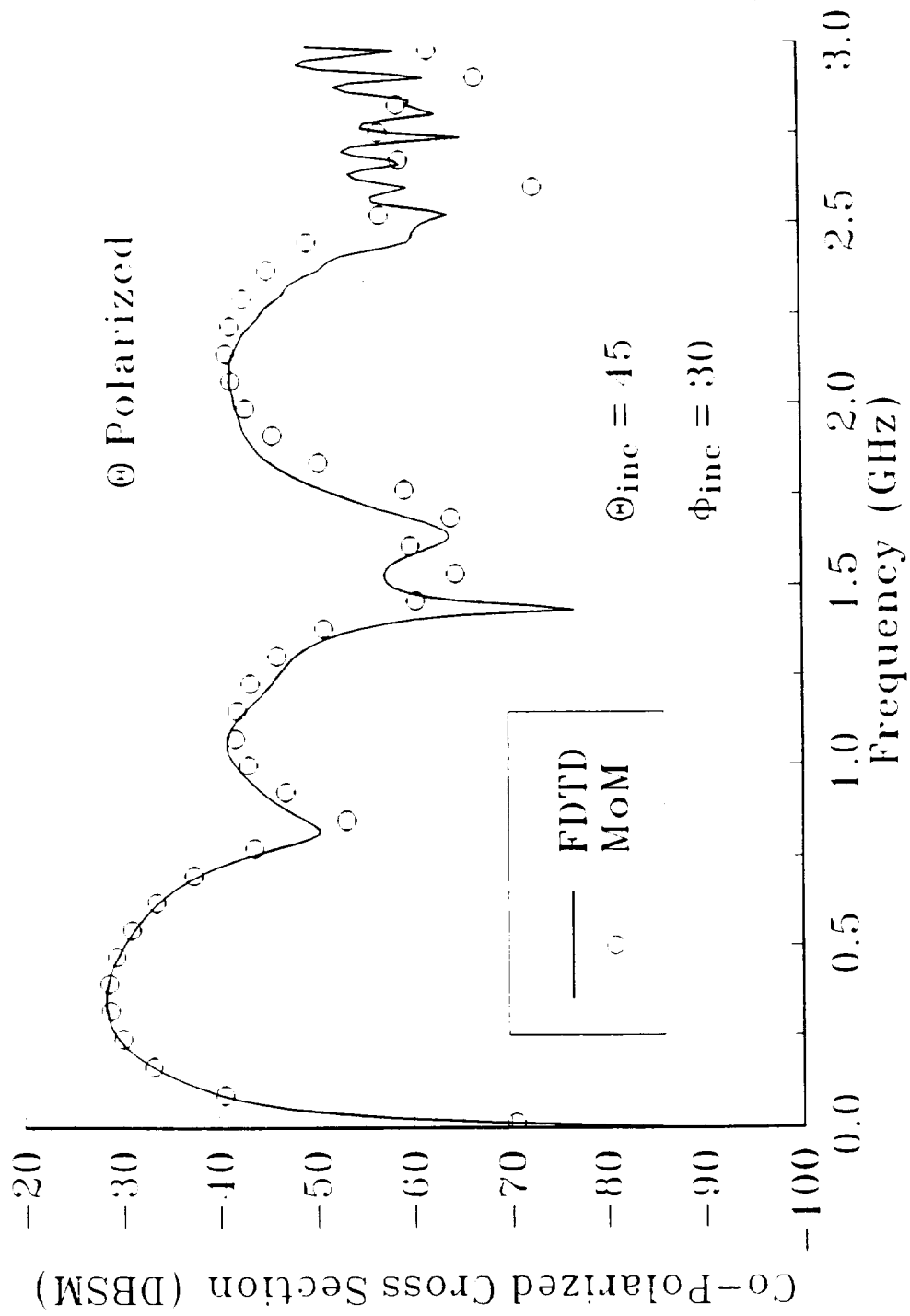
Flat Plate, 29 x 29 cm, $\sigma=0.2$ ($Z_s = 500 \Omega$)
 1 cm FDTD cells, 60 x 60 x 49 space



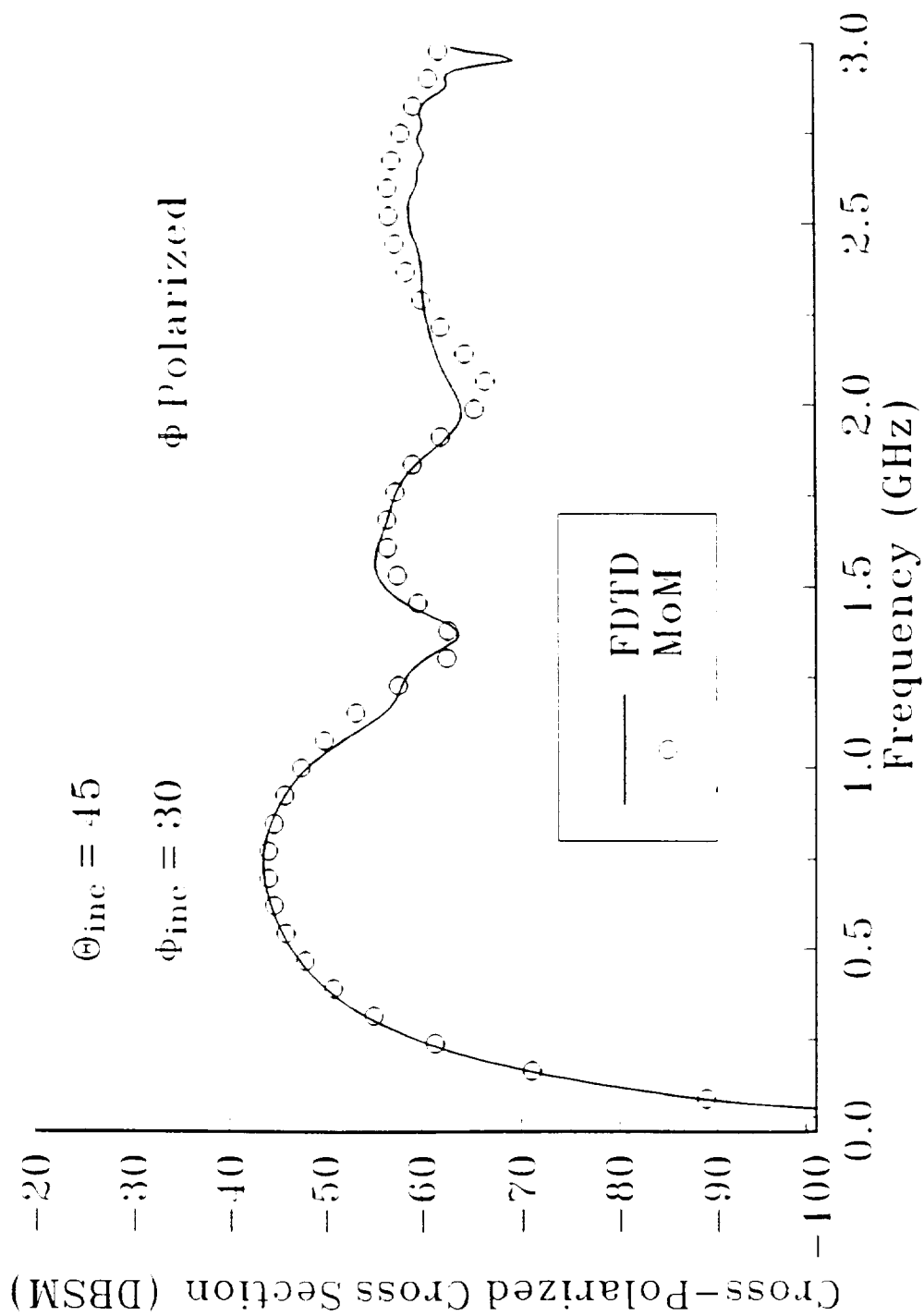
Flat Plate, 29 x 29 cm, $\sigma=0.2$ ($Z_s = 500 \Omega$)
 1 cm FDTD cells, 60 x 60 x 49 space



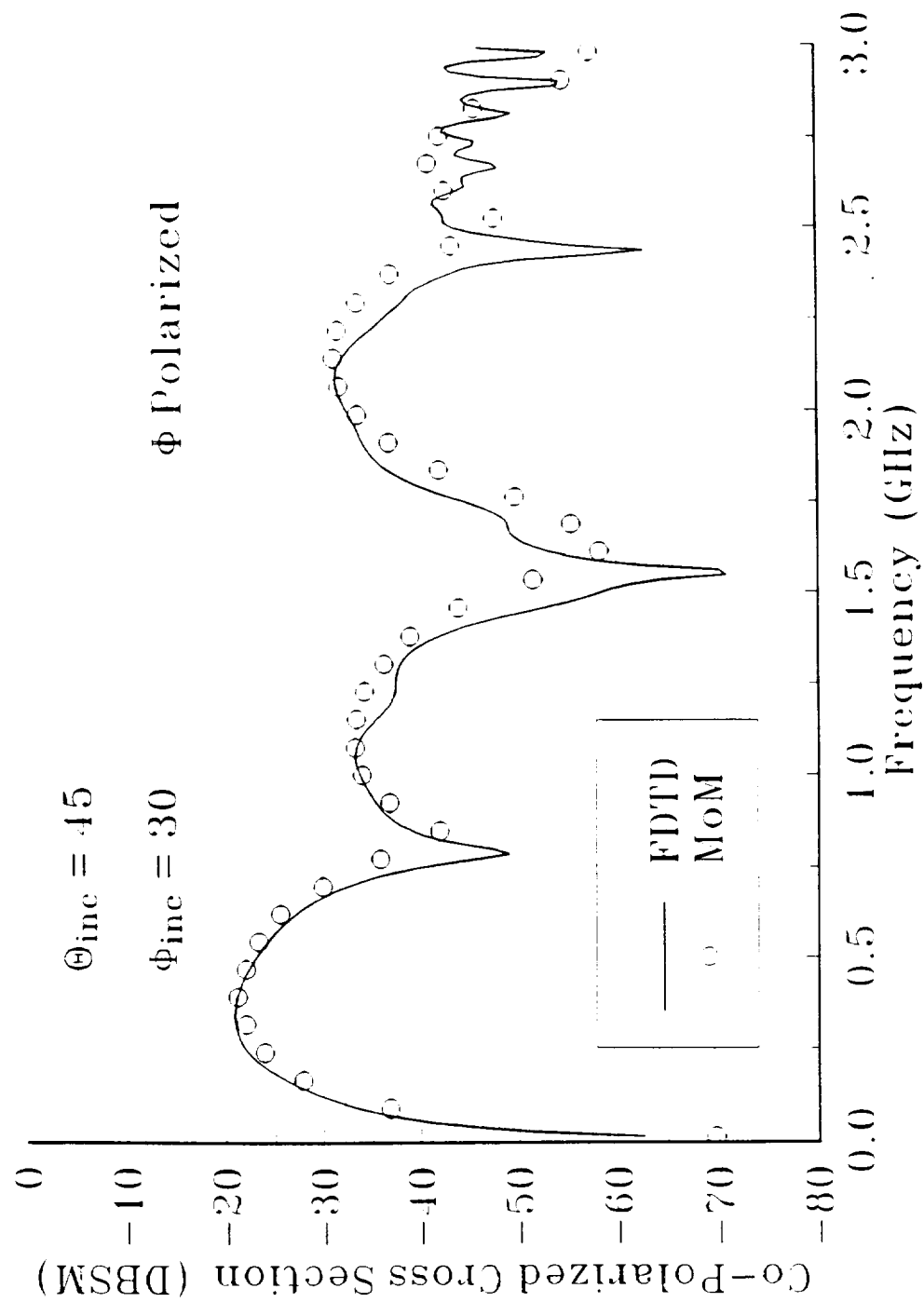
Flat Plate, 29×29 cm, $\sigma=0.2$ ($Z_s = 500 \Omega$)
 1 cm FDTD cells, $60 \times 60 \times 49$ space



Flat Plate, 29 x 29 cm, $\sigma=0.2$ ($Z_s = 500 \Omega$)
 1 cm FDTD cells, 60 x 60 x 49 space



Flat Plate, 29 x 29 cm, $\sigma=0.25$, $\epsilon_r=3.0$
 1 cm FDTD cells, 60 x 60 x 49 space



PEC Plate, 21 x 21 cm, with 4 cm 500 Ω Edge Treatment 1 cm FDTD cells, 60 x 60 x 49 space

